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Quantum-state diffusion with adaptive noise

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Abstract. We present a scheme for stochastic quantum-state diffusion (QSD) with adaptive noise to calculate the time evolution of an arbitrary observable of an open system. The method is based on the fact that the observable is much less sensitive to adaptive noise than to noise with a random phase. Hence, the individual realisations of the expectation value of the observable stay closer to the average evolution and fewer realisations are required to obtain the ensemble average. This is illustrated by applying QSD to a driven two-level system using both randomly phased and adaptive noise. Applying QSD with adaptive noise to an undriven two-level system enables us to derive a deterministic Schrödinger equation that produces the exact evolution of an arbitrary observable.

PACS. 42.50.Lc Quantum fluctuations, quantum noise, and quantum jumps – 32.80.−t Photon interactions with atoms

1 Introduction

When studying open systems, the loss of information from the system to the outside world requires the use of a density matrix $\rho(t)$ for the state of the system. Usually the system can be described by a quantum master equation of the general form [1]

$$\begin{aligned}\frac{d}{dt}\rho(t) &= -\frac{i}{\hbar}[H,\rho(t)] \\ &\quad -\frac{1}{2}\sum_k [c_k^\dagger c_k \rho(t) + \rho(t) c_k^\dagger c_k - 2c_k\rho(t)c_k^\dagger] \\ &= -\frac{i}{\hbar}[H_{\mathit{eff}}\,\rho(t) - \rho(t)H_{\mathit{eff}}^\dagger] + \sum_k c_k\rho(t)c_k^\dagger\ , \qquad (1)\end{aligned}$$

where H is the Hamilton operator for the closed system. The Lindblad operator c_k represents the coupling to the outside world through the k^{th} open channel [2] and $H_{\mathit{eff}} = H - i\hbar/2\sum_k c_k^\dagger c_k$ is an effective, non-hermitian Hamiltonian. The effect of H_{eff} is in Liouville form and does not mix an initially pure state. The last term of equation (1) cannot be written in Liouville form and it requires the use of a density matrix to describe a mixed state of the system.

In this article we focus on the evolution of observables of the system, rather than the internal state of the system itself. The expectation value $\langle A\rangle = \mathrm{tr}\rho(t)A$ of any observable A obeys the differential equation

$$\frac{d}{dt}\langle A\rangle = \frac{i}{\hbar}\langle H_{\mathit{eff}}^\dagger A - A\,H_{\mathit{eff}}\rangle + \sum_k \langle c_k^\dagger A c_k\rangle\ . \qquad (2)$$

This is generally not a closed equation and equation (1) must be solved after all.

As an alternative to equation (1) one can use a quantum-trajectory method, which requires only state vectors to be evaluated [3–5]. These quantum-trajectory techniques mainly fall into two classes, which are termed the quantum-jump method and the quantum state-diffusion method. The trajectory methods simulate the continuous observation of the information leaking out of the open channels. The information that is thus retrieved causes the system to remain pure by continuous wavefunction collapse. The outcome of the fictitious measurement is however random and one should average over all possible outcomes. This can still be numerically advantageous if the number of dimensions of the state vector, which should be squared for a density matrix, outweighs the number of trajectories that need to be averaged for a satisfactory result.

There are (infinitely) many ways to perform the fictitious measurement. Each one leads to a different ensemble with different properties for the individual trajectories. Naturally, the ensemble average is the same for all these methods. In practice, however, one cannot take the average over the entire, infinite ensemble. It would therefore be beneficial if the individual trajectories are as close to the average as possible so that few realisations are required to obtain an adequate result.

2 Linear QSD equation

We will start out from the linear quantum state-diffusion (QSD) method [6], which we briefly summarise. For simplicity we assume the system to start in a pure state,

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$\rho(0) = |\psi_0\rangle\langle\psi_0|$. An ensemble of pure state trajectories is generated with the following linear stochastic differential equation,

$$d|\psi(t)\rangle = -\frac{i}{\hbar} H_{\mathit{eff}} |\psi(t)\rangle dt + \sum_k c_k |\psi(t)\rangle d\xi_k(t) \ . \tag{3}$$

Each trajectory starts in the same initial state $|\psi_0\rangle$. The first term in equation (3) is deterministic. The second term contains the stochastic increments $d\xi_k(t)$, one for each open channel. Because of this term each trajectory will randomly differ from the other trajectories. We assume the stochastic increments $d\xi_k(t)$ to have the properties

$$\overline{d\xi_k(t)\ d\xi^*_{k'}(t)} = \delta_{k,k'} dt \qquad \text{and} \qquad \overline{d\xi_k(t)} = 0 \tag{4}$$

and

$$\overline{|\psi(t)\rangle\langle\psi(t)|\ d\xi_k(t)} = 0 \qquad \text{and}$$
$$\overline{|\psi(t)\rangle\langle\psi(t)|\ d\xi_k(t)\ d\xi^*_{k'}(t)} = \overline{|\psi(t)\rangle\langle\psi(t)|}\ \delta_{k,k'} dt \ , \tag{5}$$

where the overline notation indicates an ensemble average. The requirement (5) basically means that $d\xi_k(t)$ should be statistically independent from $|\psi(t)\rangle$ to a certain degree. The conditions (4) and (5) are sufficient to ensure that

$$\overline{|\psi(t)\rangle\langle\psi(t)|} = \rho(t) \tag{6}$$

for all $t \geq 0$. Note that the individual trajectories generally do not remain normalised.

Instead of simulating $\rho(t)$ to obtain the evolution $\langle A(t)\rangle = \mathrm{tr}\rho(t)A$ of an observable A, one can also calculate the ensemble average over single realisations $\langle A(t)\rangle_c$, *i.e.*

$$\langle A(t)\rangle = \overline{\langle A(t)\rangle_c} \ . \tag{7}$$

In order to simulate the expectation value $\langle A\rangle$, the single realisations $\langle A\rangle_c$ should be calculated with respect to the unnormalised state vector,

$$\langle A(t)\rangle_c = \langle\psi(t)|A|\psi(t)\rangle \ . \tag{8}$$

The subscript 'c' indicates that the expectation value is *conditioned* to a single realisation $\xi_k(t)$ of the noise. Using equation (3) we find for the stochastic differential of a single realisation $\langle A\rangle_c$, up to first order in dt,

$$\begin{aligned} d\langle A\rangle_c = \frac{i}{\hbar}\langle H^\dagger_{\mathit{eff}} A - A\ H_{\mathit{eff}}\rangle_c dt + \sum_k \langle c^\dagger_k A c_k\rangle_c\ |d\xi_k(t)|^2 \\ + \sum_k [\langle A c_k\rangle_c\ d\xi_k(t) + \langle c^\dagger_k A\rangle_c\ d\xi^*_k(t)] \\ + \sum_{k\neq k'} \langle c^\dagger_{k'} A c_k\rangle_c\ d\xi_k(t)\ d\xi^*_{k'}(t) \ . \end{aligned} \tag{9}$$

In equation (9) we had to evaluate the differential up to second order in $d\xi_k(t)$ in order to obtain the result up to first order in dt. This can be seen from equation (4). Of course, when the requirements (4) and (5) are fulfilled, the ensemble average of equation (9) coincides with equation (2). On the other hand, due to the presence of the linear terms in $d\xi_k(t)$ and $d\xi^*_k(t)$, a single realisation will usually produce sharp noise and the first time derivative of $\langle A(t)\rangle_c$ does not exist. Since this noise will smooth out in the ensemble average, the single realisations will deviate appreciably from the average. For numerical simulations it is advantageous if the individual realisations are already close to the average. As we shall demonstrate in the subsequent section, this can be accomplished by choosing appropriate statistics for $\xi_k(t)$.

3 Adaptive noise

The first two terms on the right-hand side of equation (9) contribute to the average evolution (2); the other terms vanish when averaged. In the special case that $|d\xi_k(t)|^2 = dt$ for each individual trajectory, the first two terms in equation (9) alone would already reproduce the differential equation (2). This would be accomplished by the choice

$$d\xi_k(t) = \sqrt{dt}\ e^{i\alpha_k(t)} \ , \tag{10}$$

with $\alpha_k(t)$ a stochastic phase. In order that condition (4) is fulfilled, $\alpha_k(t)$ must obey

$$\overline{e^{i\alpha_k(t)}} = 0 \ . \tag{11}$$

Usually one considers either randomly phased complex noise, with $\alpha_k(t)$ uniformly distributed between 0 and 2π, or real noise, with $\alpha_k(t)$ attaining the values 0 and π with equal probability. However, the requirement (11) allows for other distributions. In any case $d\xi_k(t)$ represents a random walk with infinitesimal step size. The different probability distributions for the stochastic increments can be viewed to correspond to different measurement schemes.

We consider only true observables A, which are represented by hermitian operators. Then the identity $|\langle A c_k\rangle_c| = |\langle c^\dagger_k A\rangle_c|$ holds. With the choice (10) for $d\xi_k$, the third term on the r.h.s. of equation (9) can be made to vanish by choosing α_k in such a way that

$$\langle A c_k\rangle_c\ e^{i\alpha_k(t)} + \langle c^\dagger_k A\rangle_c\ e^{-i\alpha_k(t)} = 0 \ . \tag{12}$$

In fact, this equation (12) is valid for two opposite phases α_k of the complex noise, which we specify by the two distinct values $\varphi_k(t)$ and $\varphi_k(t) + \pi$. Figure 1 schematically show these two solutions to equation (12). In order that condition (11) remains satisfied we give these two values the same probability 1/2. This means that we select a special ensemble, specified by

$$d\xi_k(t) = i\frac{\langle c^\dagger_k A\rangle_c}{|\langle c^\dagger_k A\rangle_c|} d\xi'_k(t) \qquad \text{and} \qquad d\xi'_k(t) = \pm\sqrt{dt} \ , \tag{13}$$

where the sign of $d\xi'_k(t)$ is chosen randomly with equal probability 1/2 for either sign. Although the real stochastic increments $d\xi'_k(t)$ do not depend on the state of the

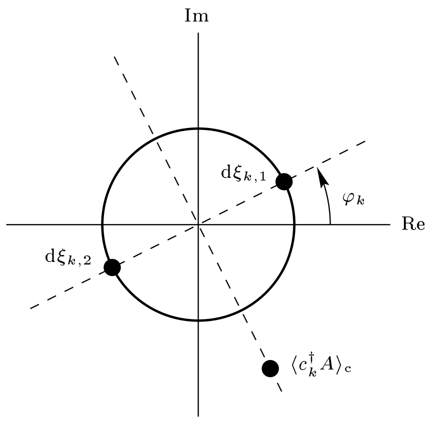

Fig. 1. Schematic construction of the adaptive phase φ_k of the stochastic increment $d\xi_k$. φ_k is chosen in such a way that $d\xi_k$ is "perpendicular" to $\langle c_k^\dagger A\rangle_c$ in the complex plane.

system, the actual complex increments $\xi_k(t)$ do depend on $|\psi(t)\rangle$. Nevertheless, the conditions (4) and (5) are obviously fulfilled so that the ensemble still reproduces the master equation (1). The noise specified by $d\xi_k$ according to equation (13) can be termed phase-adaptive noise.

The definition (13) is not defined when the denominator vanishes. However, this imposes no restriction since for such instants the third term in equation (9) will vanish regardless the choice of the phase angle α_k.

The resulting stochastic differential equation for the state vector $|\psi(t)\rangle$ takes the form

$$d|\psi(t)\rangle = -\frac{i}{\hbar}H_{\mathit{eff}}|\psi(t)\rangle dt + i\sum_k \frac{\langle c_k^\dagger A\rangle_c}{|\langle c_k^\dagger A\rangle_c|}c_k|\psi(t)\rangle d\xi_k'(t)\ . \tag{14}$$

This stochastic equation, with $d\xi_k'(t) = \pm\sqrt{dt}$, specifies the QSD equation with phase-adaptive noise. The trajectories generated by this equation are continuous, but they do not have a first time derivative. The realisations $\langle A\rangle_c$, on the other hand, *do* have a first time derivative, *i.e.* they are smooth. The fluctuations only appear as higher order noise. We postpone the discussion on the interpretation of equation (14) to Section 6.

We like to note that similar results can be obtained starting with certain non-linear QSD equations such as the equation originally proposed in reference [4]. However, the considerations just presented are much more transparent starting with the linear QSD equation. Moreover, some of the non-linear methods require $\overline{d\xi_k(t)^2} = 0$, which is violated by the adaptive noise. Therefore such methods cannot easily serve as a starting point for the derivation of an equation with adaptive noise.

The original, nonlinear, QSD equation has the advantage that each individual trajectory remains normalised. This ensures that each realisation $\langle A\rangle_c$ stays within the physically interpretable range. The nonlinear QSD equation, which can be derived directly from the physical measurement scheme of homodyne detection [7], is given by [4]

$$d|\psi(t)\rangle = \Big(-\frac{i}{\hbar}H_{\mathit{eff}} + \langle c_k^\dagger\rangle_c c_k - \tfrac{1}{2}\langle c_k^\dagger\rangle_c\langle c_k\rangle_c\Big)|\psi(t)\rangle dt + \sum_k \big(c_k - \langle c_k\rangle_c\big)|\psi(t)\rangle d\xi_k(t)\ . \tag{15}$$

Using the same considerations as for the linear equation, we derive for the adaptive noise

$$d\xi_k(t) = i\frac{\langle c_k^\dagger A\rangle_c - \langle c_k^\dagger\rangle_c\langle A\rangle_c}{|\langle c_k^\dagger A\rangle_c - \langle c_k^\dagger\rangle_c\langle A\rangle_c|}d\xi_k'(t)\ . \tag{16}$$

As before, this noise does not affect the realisation $\langle A\rangle_c$ to first order in $d\xi_k(t)$.

4 Driven two-level system

As a demonstration of QSD with adaptive noise we apply it to a well known configuration and compare the results with those obtained using randomly phased noise. As a model system we take the driven two-level atom. All interactions can be written in terms of the unit operator and the three components of the spin-vector operator **S**. These we define in the standard way as

$$S_x = \tfrac{1}{2}(S + S^\dagger)\ ,\quad S_y = \tfrac{1}{2}i(S - S^\dagger) \quad \text{and} \quad S_z = \tfrac{1}{2}(S^\dagger S - SS^\dagger)\ , \tag{17}$$

where the lowering operator

$$S = |g\rangle\langle e| \tag{18}$$

simply transfers the excited state $|e\rangle$ to the ground state $|g\rangle$. The system has only one open channel, the decay from the excited state to the ground state with a rate Γ. We describe the density matrix with respect to a frame rotating with frequency ω of the driving field. The resulting master equation is

$$\frac{d}{dt}\rho(t) = -\frac{i}{\hbar}[H_{\mathit{eff}}\rho(t) - \rho(t)H_{\mathit{eff}}^\dagger] + \Gamma S\rho(t)S^\dagger\ , \tag{19}$$

with the effective Hamiltonian

$$H_{\mathit{eff}} = -\hbar\Delta S_z - \hbar\Omega S_x - i\hbar\frac{\Gamma}{2}S^\dagger S\ , \tag{20}$$

where $\Delta = \omega - \omega_0$ is the detuning of the driving frequency with respect to the atomic transition frequency and Ω is the Rabi frequency. As the initial state of the system we choose the excited state, so $\rho(0) = |e\rangle\langle e|$. The system, as it evolves according to equation (19), undergoes a Rabi oscillation between the excited and the ground state because of the driving field, while the decaying terms try to relax the system to the ground state. As a result of the latter the system ends up in a mixed steady-state. The final excitation depends on the parameters Δ, Γ and Ω.

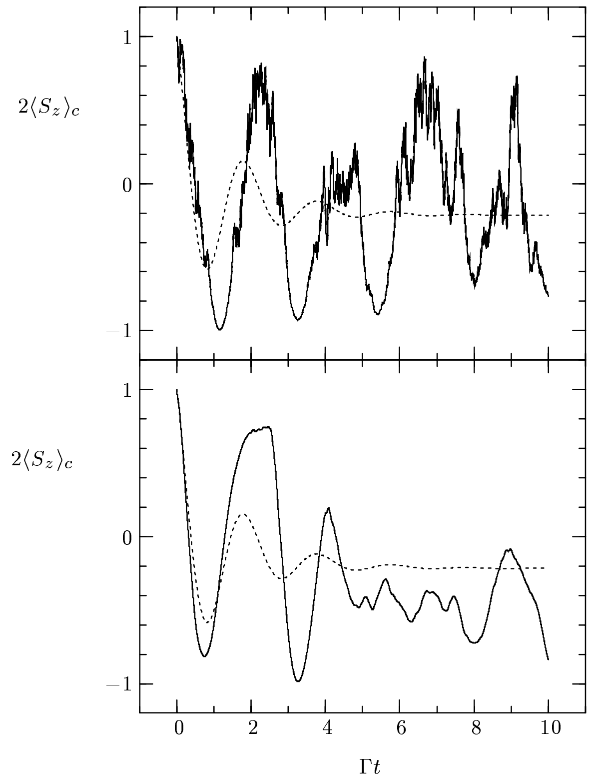

Fig. 2. Typical realisations (solid curves) of the inversion $2\langle S_z\rangle_c$ plotted as a function of the scaled time Γt (upper graph) using randomly phased noise and (lower graph) using adaptive noise. The parameters are $\Delta = \frac{1}{3}\Omega = \Gamma$. The dashed curves show the exact average evolution $2\langle S_z\rangle$, obtained by direct integration of equation (19).

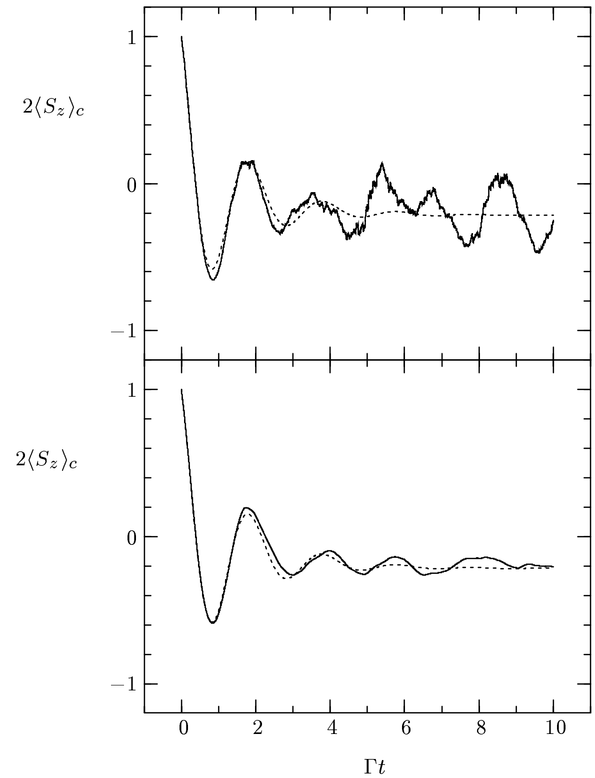

Fig. 3. Ensemble averages over 100 realisations of the inversion $2\overline{\langle S_z\rangle_c}$ plotted as a function of the scaled time Γt (upper graph) using randomly phased noise and (lower graph) using adaptive noise. The parameters are $\Delta = \frac{1}{3}\Omega = \Gamma$. The dashed curves show the exact average evolution $2\langle S_z\rangle$, obtained by direct integration of equation (19).

The dashed curves in Figure 2 show the time evolution of the inversion, which is simply $2\langle S_z\rangle$, for a particular choice of the parameters.

The nonlinear QSD-evolution is given by (15)

$$d|\psi(t)\rangle = \Big(-\frac{i}{\hbar}H_{\mathit{eff}} + \Gamma\langle S^\dagger\rangle_c S - \tfrac{1}{2}\Gamma\langle S^\dagger\rangle_c\langle S\rangle_c\Big)|\psi(t)\rangle dt + \sqrt{\Gamma}\big(S - \langle S\rangle_c\big)|\psi(t)\rangle d\xi(t) \qquad (21)$$

with $|\psi(0)\rangle = |e\rangle$. First we will choose randomly phased noise. Every individual trajectory will essentially display the same behaviour as the average evolution does. However, since the trajectories remain in a pure state, there will be no relaxation to a steady state, which has to be a mixed state due to the incoherent decay channel. Because of the noise, which accumulates in time, the Rabi oscillations of the trajectories will dephase with respect to each other. Of course, a full ensemble average will yield results identical to those obtained with the master equation. The upper graph of Figure 2 shows a typical single realisation of the inversion using randomly phased noise. The individual realisations are not smooth, *i.e.* they do not have a first time derivative.

Now we will apply adaptive noise with respect to the observable S_z,

$$d\xi(t) = i\frac{\langle S^\dagger S_z\rangle_c - \langle S^\dagger\rangle_c\langle S_z\rangle_c}{|\langle S^\dagger S_z\rangle_c - \langle S^\dagger\rangle_c\langle S_z\rangle_c|}d\xi'(t)\ . \qquad (22)$$

The lower graph of Figure 2 shows a typical single realisation of the inversion using this adaptive noise. This time each of the realisations is smooth. As a result, the realisations stay closer to the average than the realisations based on randomly phased noise. This is illustrated in Figure 3 where the average over 100 realisations is plotted for both types of noise together with the exact result. Clearly, the adaptive-noise method displays a better convergence.

The first time derivative of the realisation $\langle A\rangle_c$, given by equation (9), using adaptive noise is only equal to the average $\langle A\rangle$, given by equation (2), when the state of the system is pure, *i.e.* when $\rho = |\psi\rangle\langle\psi|$. Assuming that this is the case at time $t = 0$, the slope of $\langle A\rangle_c$ coincides with the average initially, but not at later times when the individual trajectory $|\psi\rangle$ no longer represents the average state. An alternative way to understand the deviation from the average at later times is the fact that the higher-order time derivatives of $\langle A\rangle_c$ are not equal to the average; the noise *does* contribute to higher-order differentials such as $d(d\langle A\rangle_c)$, even for adaptive noise. This is the reason for labelling the resulting noise in $\langle A\rangle_c$ "higher order" noise. The effect of adaptive noise is more like a drift; the deviation from the average takes place on a longer time scale than in the case of true diffusion due to randomly phased noise.

Only when the higher order noise terms disappear as well is the realisation $\langle A\rangle_c$ identical to the average $\langle A\rangle$. This is the case, for example, when $A = 1$. The use of adapting to the expectation value $\langle 1\rangle$ is mainly that it shows that for $A = 1$ the prescription (14) produces normalised trajectories. A more interesting example is studied in the next section.

5 Undriven two-level system

Now we present the results of adaptive noise applied to the undriven two-level system. This system is described by the same equation (19) as the driven system, but with $\Omega = \Delta = 0$. The resulting master equation is

$$\frac{d}{dt}\rho(t) = -\frac{\Gamma}{2}[S^\dagger S\rho(t) + \rho(t)S^\dagger S - 2S\rho(t)S^\dagger] \; . \qquad (23)$$

The linear QSD equation using adaptive noise with respect to an observable A is given by (3)

$$d|\psi(t)\rangle = -\frac{\Gamma}{2}S^\dagger S|\psi(t)\rangle dt + i\sqrt{\Gamma}\frac{\langle S^\dagger A\rangle_c}{|\langle S^\dagger A\rangle_c|}S|\psi(t)\rangle d\xi'(t) \; . \qquad (24)$$

As before, the lowest-order noise in $\langle A\rangle_c$ is zero because of the adaptive noise. In this — very special — case, however, there are no higher order noise contributions to $\langle A\rangle_c$ either. This is due to the fact that

$$S^\dagger S^\dagger = SS = 0 \qquad (25)$$

for two-level systems and that there is no driving field to re-excite the population transferred to the ground state by the stochastic evolution. The realisation $\langle A\rangle_c$ is therefore entirely deterministic and the same for every individual trajectory. Since we are generating an ensemble that reproduces the correct average results, each realisation is identical to the ensemble average,

$$\langle A\rangle = \langle A\rangle_c \; . \qquad (26)$$

We can therefore choose an arbitrary — even non-stochastic — realisation for $d\xi'(t)$. It is convenient to choose "alternating noise", *i.e.* the sign of the noise in equation (24) at any instant is opposite to the sign at the previous time step. Evaluating the differential $d|\psi(t)\rangle$ up to second order in $\sqrt{dt}$, using once a minus and once a plus sign, makes the first order terms in $\sqrt{dt}$ cancel. Up to first order in dt we obtain

$$\frac{d}{dt}|\psi_A(t)\rangle = -\frac{\Gamma}{2}\left(S^\dagger S - \frac{\langle S^\dagger AS\rangle_c}{\langle AS\rangle_c}S\right)|\psi_A(t)\rangle \; . \qquad (27)$$

Here we have written $|\psi_A\rangle$ instead of $|\psi\rangle$ since only the expectation value for the observable A is correct. Equation (27) no longer generates an ensemble. It is a non-linear, Schrödinger-type equation for a state vector that gives exactly the same evolution for $\langle A\rangle$ as the density matrix does. This means that only a single state-vector evolution has to be calculated for this open system that can otherwise only be described by a density matrix or an ensemble of stochastic state-vector trajectories. Note that in general $|\psi_A\rangle$ is not normalised, but still $\langle\psi_A|A|\psi_A\rangle \equiv \langle A\rangle_c = \langle A\rangle$ is exact.

One point that deserves attention is the denominator in equation (27). Of course it results directly from the denominator in equation (13). In Section 3 this was resolved by simply selecting a random phase in the special case that the denominator is zero. In the deterministic equation (27) this is not an option. The denominator is zero when the atom is either in the ground or in the excited state. When the atom is in the ground state, the singular term simply vanishes because the action of S on $|g\rangle$ is already zero. In the excited state, the singularity can be interpreted as an indication of a symmetric instability. Symmetry breaking by quantum noise is essential in this case. There is no deterministic way to prescribe the direction in which the dipole moment should topple over in order for the atom to decay.

6 Discussion

Although is might seem that equation (14) is a different QSD equation than equation (3), it is in fact a special implementation of the latter. The adaptive noise utilises the freedom in the choice of the actual realisation per trajectory of the noise, which only has to fulfil the statistical requirements of equation (4). Only the properties of the noise are altered compared with conventional methods. Otherwise, the evolutionary equation that generates the trajectories remains the same.

Existing diffusion methods can be equipped with adaptive noise to improve the numerical efficiency. This has been illustrated with the simple example of a driven two-level atom in Section 4. However, the method is generally applicable. Recently, we have applied the adaptive noise method to a more complex system of practical importance [8]. In that article we studied the final temperature of an atom beam, both confined and cooled by laser light in the transverse direction. Using adaptive noise only 20 trajectories were required to obtain a useful average, which is approximately 20 times more efficient than conventional methods based on randomly or fixed phased noise.

Besides the numerical advantages, the adaptive noise method can provide analytical results that cannot be obtained directly using conventional methods. We have illustrated this in Section 5, where a deterministic expression was found for a single trajectory that produces the exact average result. In earlier work we have used this to find analytical expressions for the time evolution of observables of less trivial systems [9]. Furthermore, the special case where the adaptive noise is undetermined indicates the existence of critical points, where the symmetry breaking due to quantum noise is essential.

Finally we would like to address the physical measurement scheme that would lead to adaptive noise. For an overview of several quantum trajectory methods and their corresponding measurement schemes see reference [10]. As a starting point for this discussion we take the nonlinear QSD method of equation (15). This method results from a homodyne mixing scheme, where the output from the system is mixed with a strong classical oscillator of which the phase is fixed. The noise in one of the quadrature components resulting from the mixing leads to real Gaussian white noise for $\xi(t)$. The effects of a time dependent phase of the oscillator were studied in [11]. Adaptive noise can be seen as resulting from an infinitely fast feedback of the measurement outcomes on this oscillator phase. The feedback device knows the present state of the system and precalculates the effect on the observable of interest for all possible oscillator phases and measurement outcomes. It then adjusts the oscillator phase so that the effect of any possible noise measurement is minimal. The next measurement outcome again results in a well known state and a new feedback cycle is started.

One should realise that the measurement is performed on the output field and not on the system itself. The actually measured observable of the output field will fluctuate as a result of the measurement. However, a complementary observable will not be influenced since no information about it is retrieved. One could simply state that a measured observable will fluctuate due to the measurement and a complementary observable will not. The output field is of course generated by the system and there is a (nontrivial) connection between the system and the output field observables. If we want to efficiently calculate the system observable A, as we did in this article, then the measurement should be performed on the field observable complementary to the field observable that corresponds to A. Descriptively one could say: in order to *calculate* A, one should *not measure* A. On the other hand, prerequisite for the system to be described by a pure state as a result of continuous wavecollapse is a measurement, but it should *not* be performed on A. The adaptive noise method is in fact an implementation of this principle.

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